An Example Proof By Strong Mathematical Induction

Define the sequence (c_n) as follows:

$c_0 = 2, \quad c_1 = 2, \quad c_2 = 6,$

$c_k = 3\,c_{(k-3)}$ for all integers $k \geq 3$.

To Prove: For all integers $n \geq 0$, c_n is even.

Proof: [By STRONG MATH'L INDUCTION]

[BASIS STEP]

Let $n = 0$. $\therefore c_n = c_0 = 2 = 2 \cdot 1$, so, c_0 is even.

Let $n = 1$. $\therefore c_n = c_1 = 2 = 2 \cdot 1$, so, c_1 is even.

Let $n = 2$. $\therefore c_n = c_2 = 6 = 2 \cdot 3$, so, c_2 is even.

[END OF BASIS STEP]

[Inductive Step]

Let k be any integer such that $k \geq 2$.

[Inductive Hypothesis:]

Suppose c_m is even, for every integer m such that $0 \leq m \leq k$.

[We N.T.S. that c_{k+1} is even.]

Since $k \geq 2$, $k+1 \geq 3$ and so also $k-2 \geq 0$.

$\therefore 0 \leq k-2 \leq k$.

Since $k+1 \geq 3$, $c_{(k+1)} = 3\,c_{(k-2)}$ by definition of c_t when $t \geq 3$.

Since $0 \leq k-2 \leq k$, $c_{(k-2)}$ is even by the Induction Hypothesis.

$\therefore c_{(k-2)} = 2s$ for some integer s, by def'n of "even".

$\therefore c_{(k+1)} = 3(2s)$ by substitution,

$= 2(3s)$

$= 2l$, where $l = 3s$, which is an integer.

$\therefore c_{(k+1)}$ is even.

$\therefore$ For all integers $k \geq 2$, if c_m is even, for all integers m such that $0 \leq m \leq k$, Then $c_{(k+1)}$ is even, By Direct Proof.

[END of Induction Step]

$\therefore$ For all integers $n \geq 0$, c_n is even by The Principle of Strong Mathematical Induction.

QED